Predictive wave-front correction. Use with pyramids.

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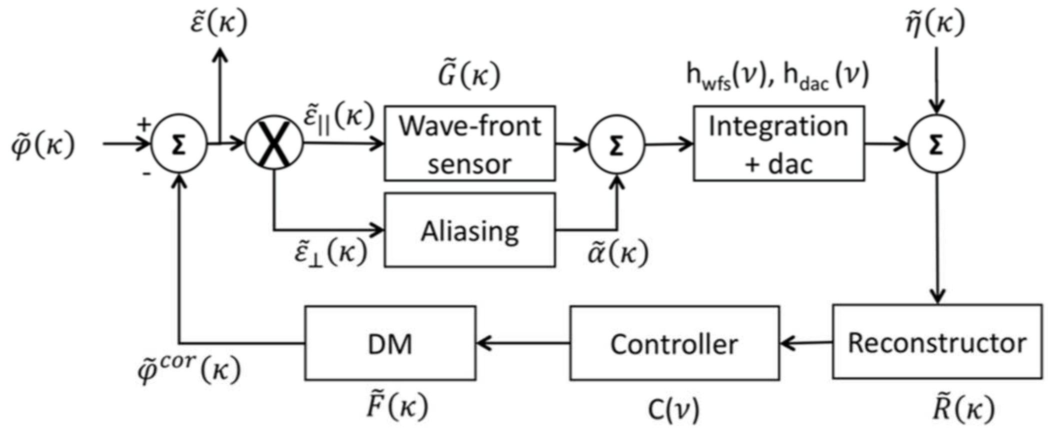
Rationale

- Initial motivation: rejection of aliasing “post-facto” using an alternative to *optical spatial filters*
- ++ Embed time-progression models into controller
  - Use of predictive control can significantly improve AO rejection close to PSF core
- Use synthetic modelling for error budget break-down
- Discretise into complex exponentials coefficients (=Fourier control) and come up with a fast real-time algorithm

Synthetic Modelling of AO

- Evaluate spatial frequency) error functions

$$\begin{bmatrix} \tilde{\varepsilon}_{||}(\boldsymbol{\kappa}) \\ \tilde{\varepsilon}_{\perp}(\boldsymbol{\kappa}) \end{bmatrix} = \begin{bmatrix} \widetilde{\boldsymbol{\varphi}}_{||}(\boldsymbol{\kappa}) \\ \widetilde{\boldsymbol{\varphi}}_{\perp}(\boldsymbol{\kappa}) \end{bmatrix} - \begin{bmatrix} \widetilde{\boldsymbol{\varphi}}_{||}^{\mathrm{cor}}(\boldsymbol{\kappa}) \\ \widetilde{\boldsymbol{\varphi}}_{\perp}^{\mathrm{cor}}(\boldsymbol{\kappa}) \end{bmatrix}$$



- Use PSDs of phase and errors in closed-loop

$$\left\langle |\widetilde{\boldsymbol{\varphi}}_{\|} - \widehat{\widetilde{\boldsymbol{\varphi}}}_{\|}|^2 \right\rangle = \left(1 + |\widetilde{\mathcal{R}}\widetilde{\mathcal{G}}|^2 \bar{\mathbf{H}}_2 - \widetilde{\mathcal{R}}\widetilde{\mathcal{G}}\bar{\mathbf{H}}_1 - (\widetilde{\mathcal{R}}\widetilde{\mathcal{G}}\bar{\mathbf{H}}_1)^*\right) \mathbf{W}_\varphi$$
$$= \left(1 + |\widetilde{\mathcal{R}}\widetilde{\mathcal{G}}|^2 \bar{\mathbf{H}}_2 - \Re\{\widetilde{\mathcal{R}}\widetilde{\mathcal{G}}\bar{\mathbf{H}}_1\}\right) \mathbf{W}_\varphi$$

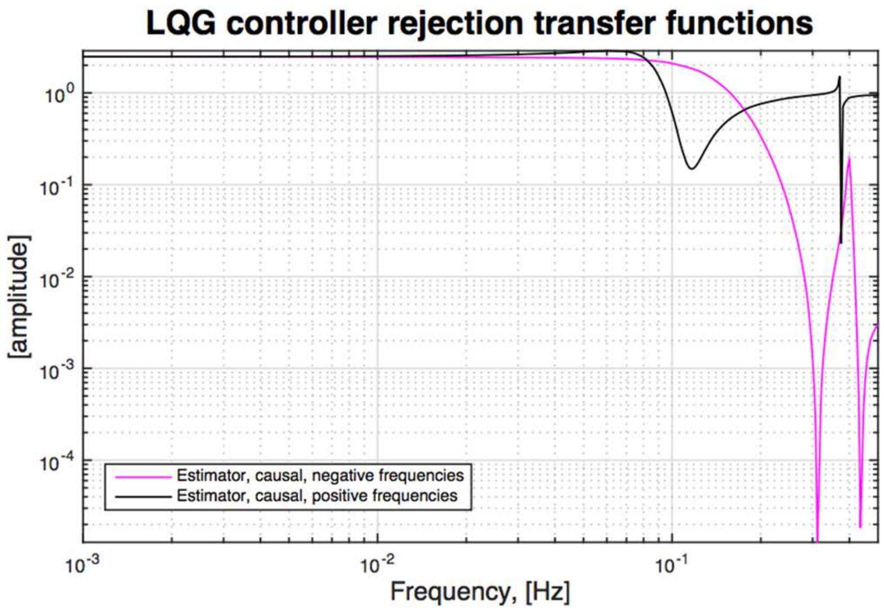
Aniso-servo error

$$\mathbf{W}_{\mathrm{RA}} = \widetilde{\mathcal{P}} \sum_{\mathbf{m} \neq 0} \left| \widetilde{\mathcal{R}}(\boldsymbol{\kappa}) \bar{\mathbf{H}}_1(\boldsymbol{\kappa}) \widetilde{\mathcal{G}}(\boldsymbol{\kappa} + \mathbf{m}/d) \right|^2 \mathbf{W}_\varphi(\boldsymbol{\kappa} + \mathbf{m}/d)$$

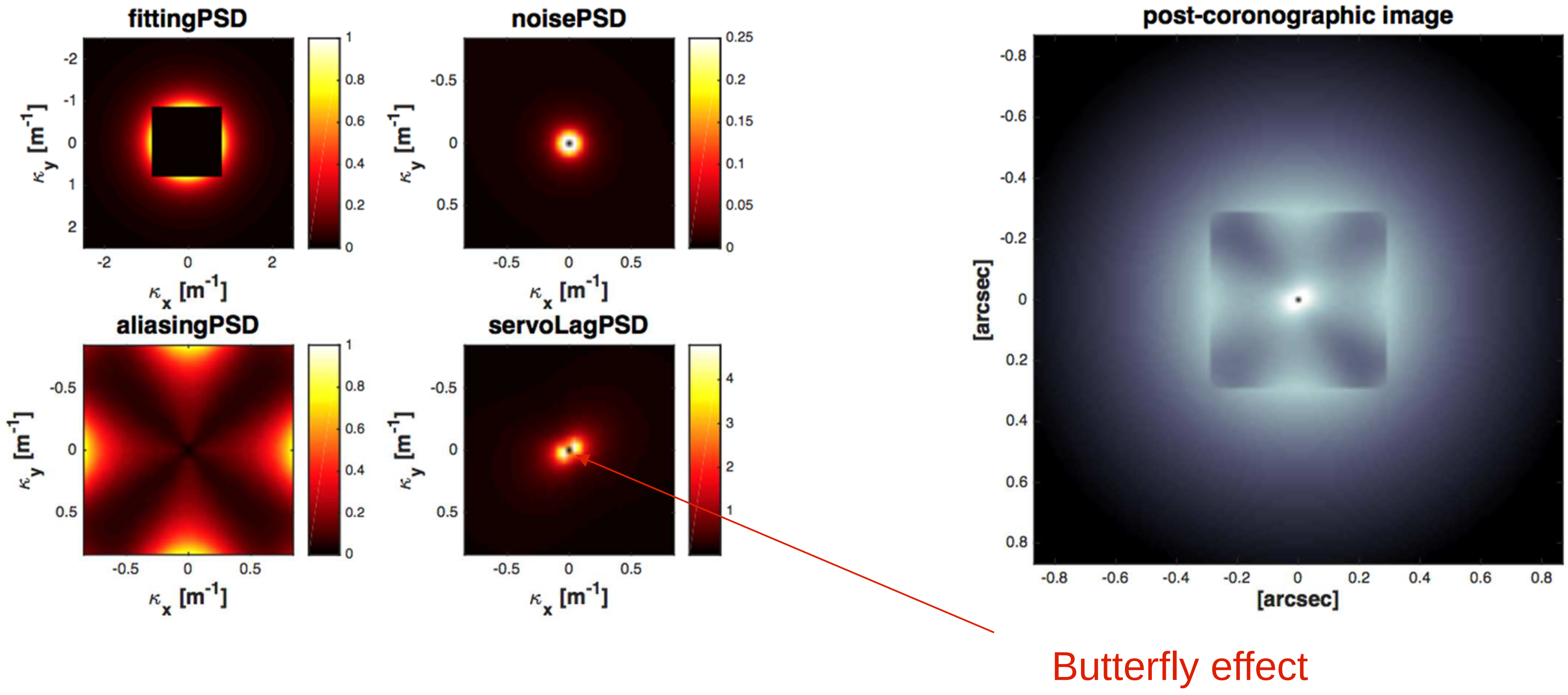
Propagated aliasing

$$\mathbf{W}_\eta = \left\langle \widetilde{\mathcal{P}} \left| \widetilde{\mathcal{R}} \widetilde{\boldsymbol{\eta}} \bar{\mathbf{H}}_\eta(\boldsymbol{\kappa}) \right|^2 \right\rangle = \widetilde{\mathcal{P}} \left| \widetilde{\mathcal{R}} \right|^2 \sigma_\eta^2 d^2 p$$

Noise propagated

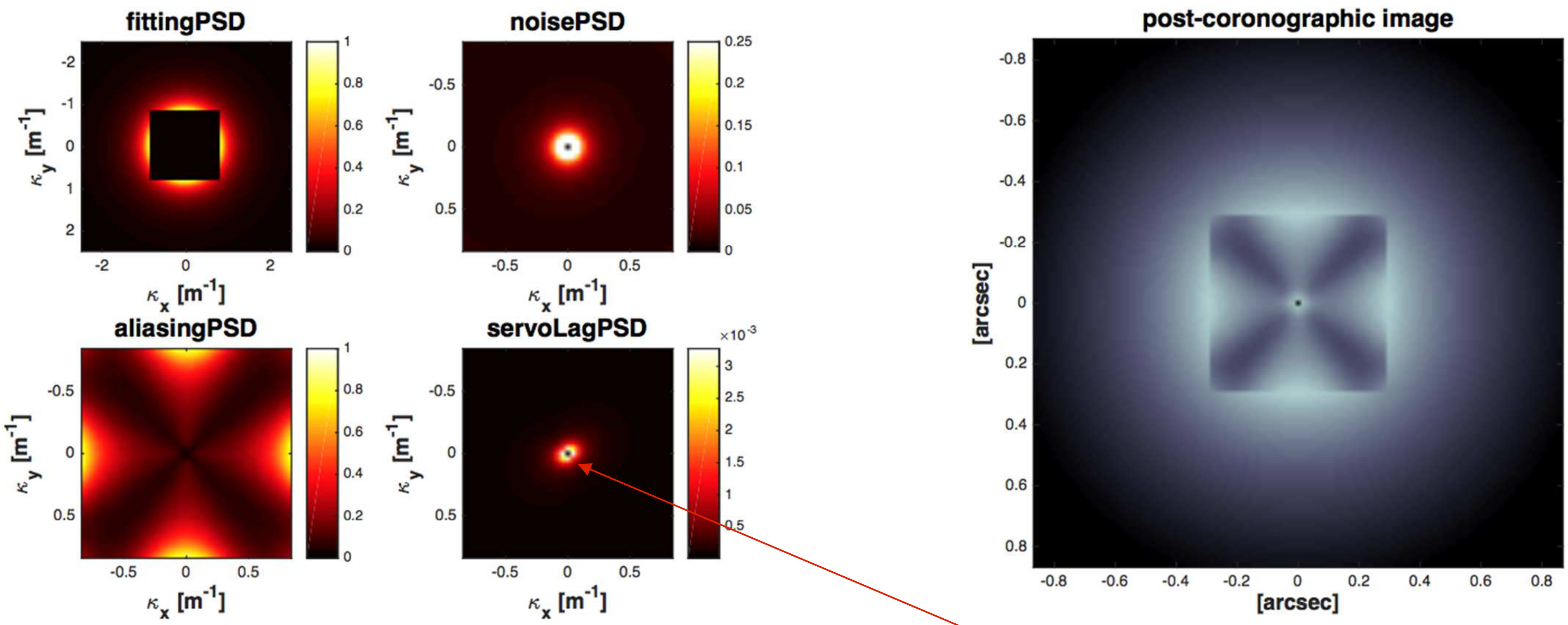


Synthetic Modelling of AO



(a) Integrator controller with a least-squares filter.

Synthetic Modelling of AO

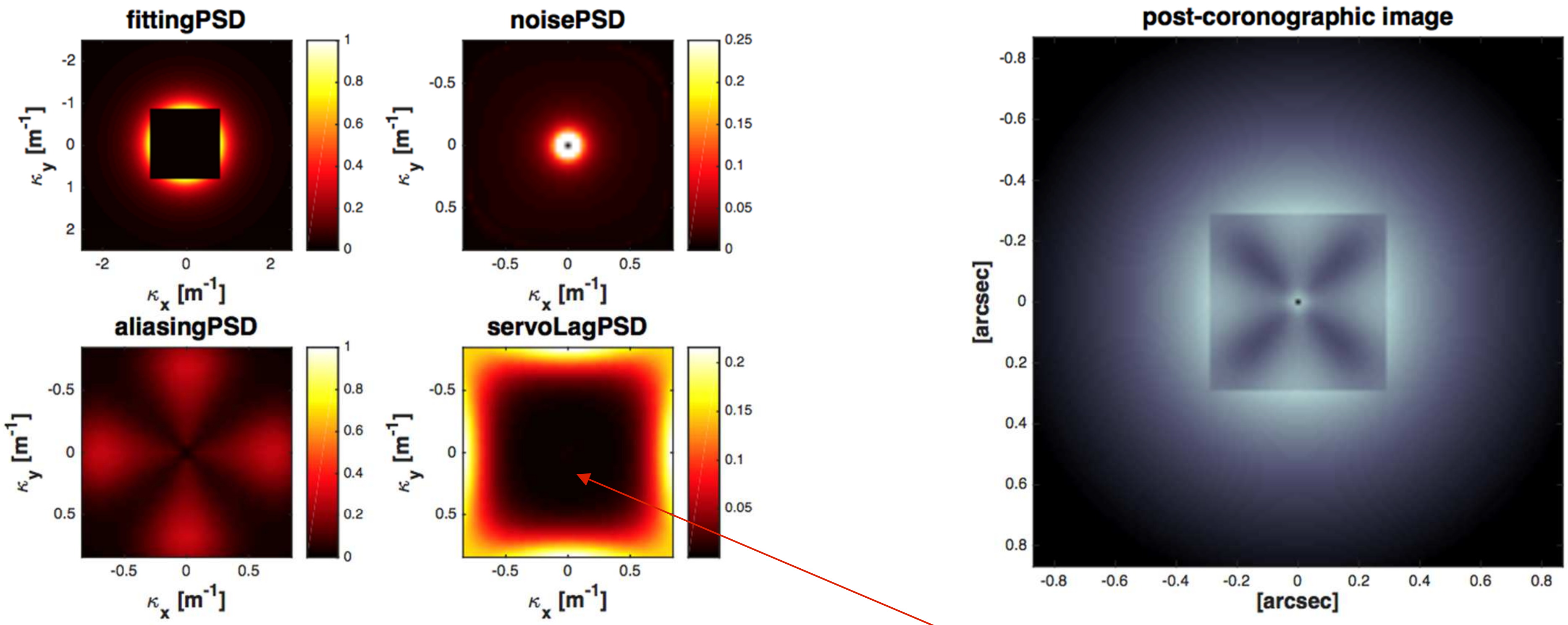


(b) DKF controller coupled to a least-squares filter.

Butterfly effect removed (under perfect wind knowledge)

C. M. Correia et al, Atelier ITHD @ LAM, May 2017

Synthetic Modelling of AO



(c) DKF controller coupled to an anti-aliasing filter.

Butterfly effect traded for aliasing => improved contrast in certain regions close to core

Pyramid filters (cross-section)

- Use physical-optics description of the pyramid
  - Fourier optics and distribution theory applied to pyramid wavefront sensors, R. Conan, 2003
- Analytic pyramid filter: Infinite telescope diameter, circular modulation, linear approximation, cross-terms dropped out, discrete measurement taken into account…

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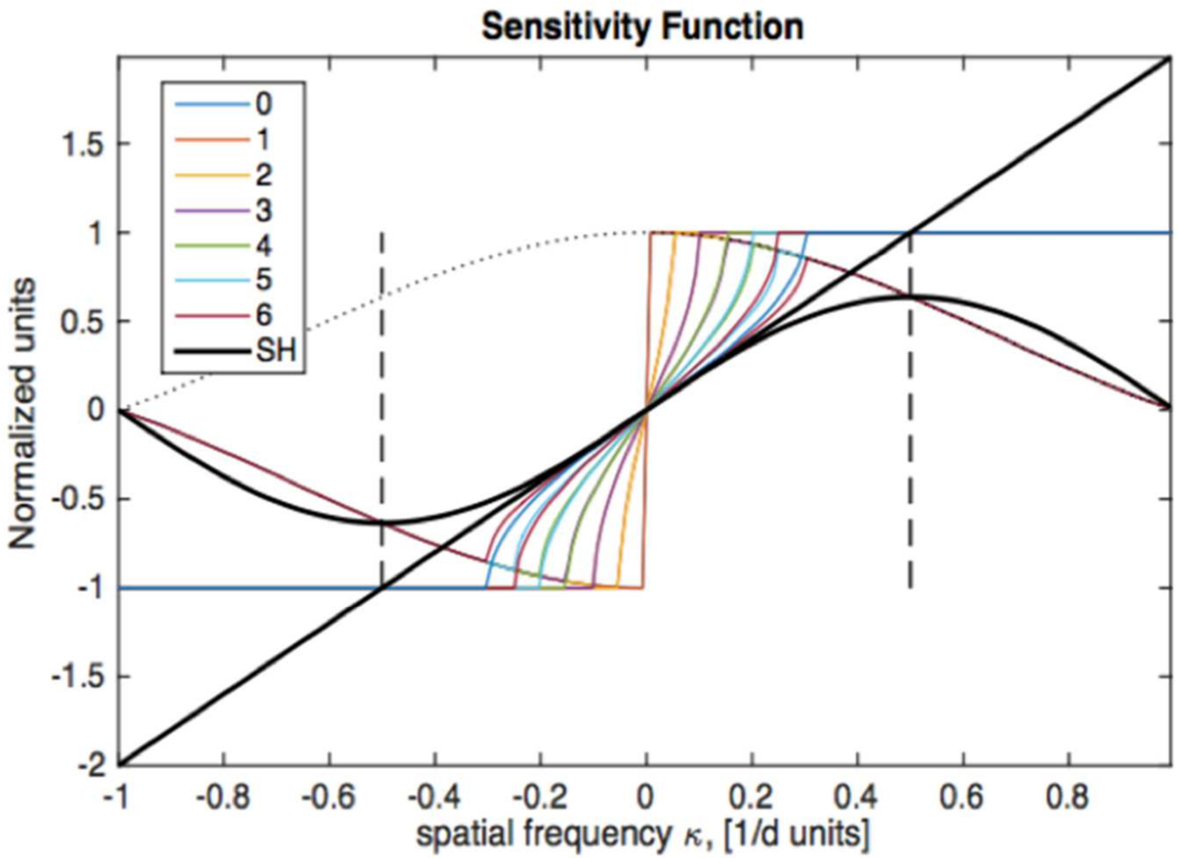
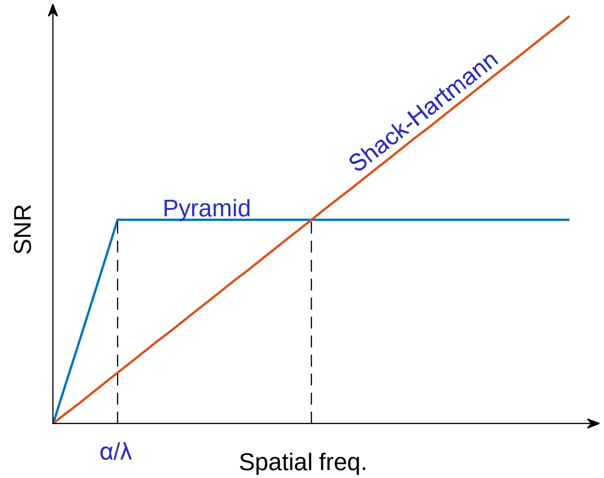
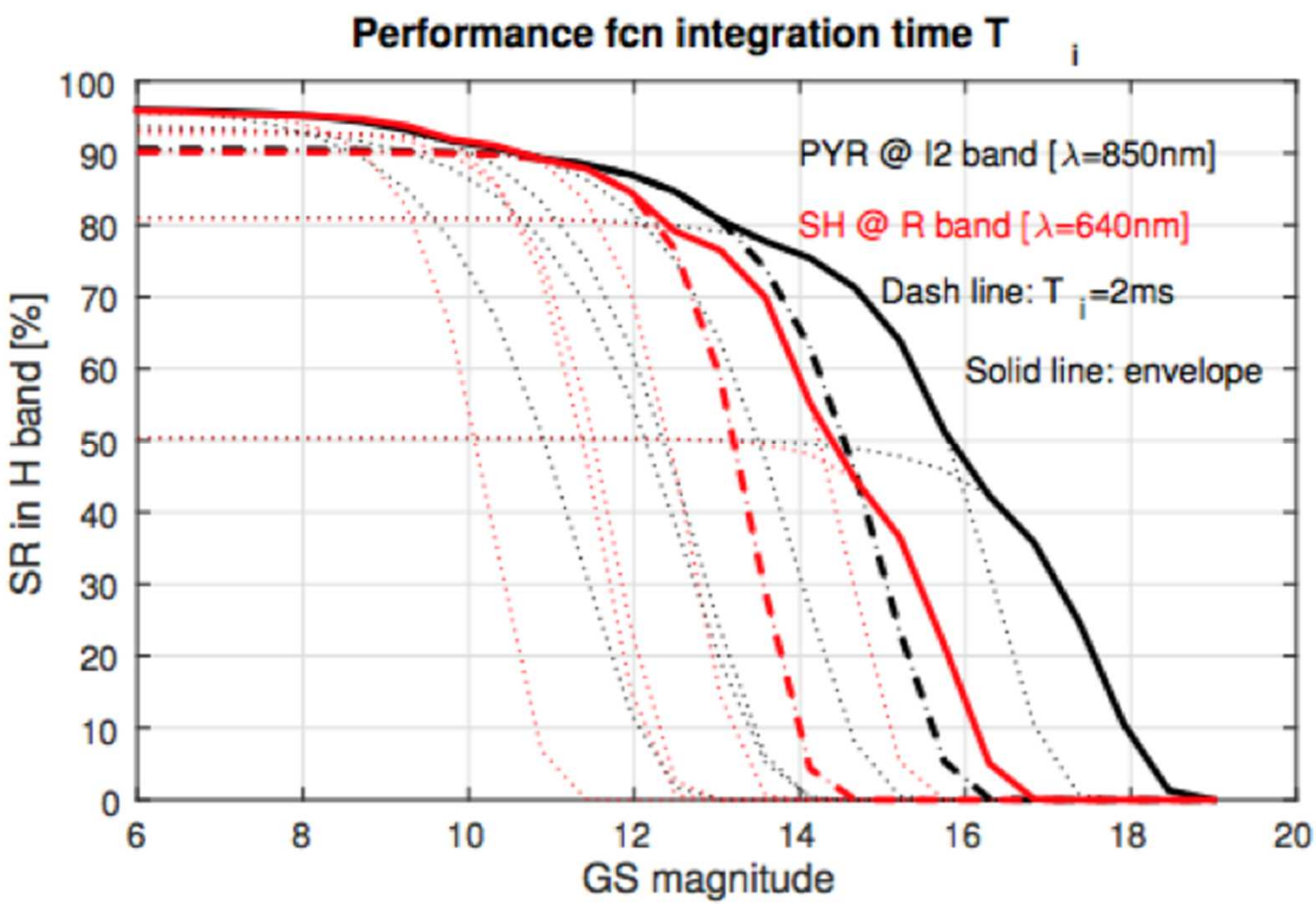




Flat response attenuated by pixel averaging

Error budgeting



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Wave-front reconstruction in the Fourier domain

Direct space		Fourier space
convolution in direct space	→	multiplication in Fourier space

$$s(\mathbf{x}) = G\phi(\mathbf{x}) + \eta(\mathbf{x})$$

$$\tilde{s}(\kappa) = \tilde{G}\tilde{\phi}(\kappa) + \tilde{\eta}(\kappa)$$

$$\hat{\phi}(\mathbf{x}) = G^{-1}s(\mathbf{x})$$

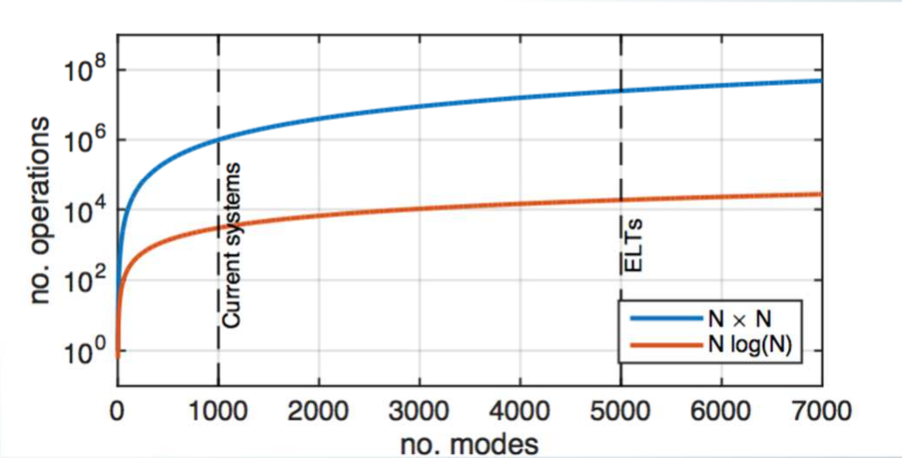
- G^\dag since inverse does not exist

$$\tilde{R}_{x/y} = \frac{\tilde{G}^*_{x/y}}{|\tilde{G}_x|^2 + |\tilde{G}_y|^2 + \gamma\frac{W_n}{W_\phi}}$$

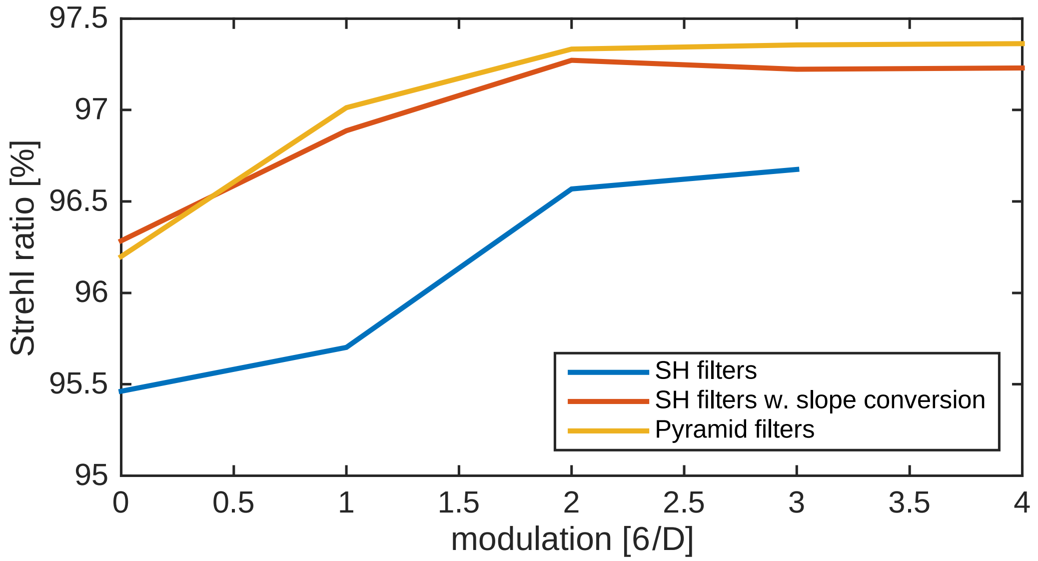
N correctable modes → **N×N operations**

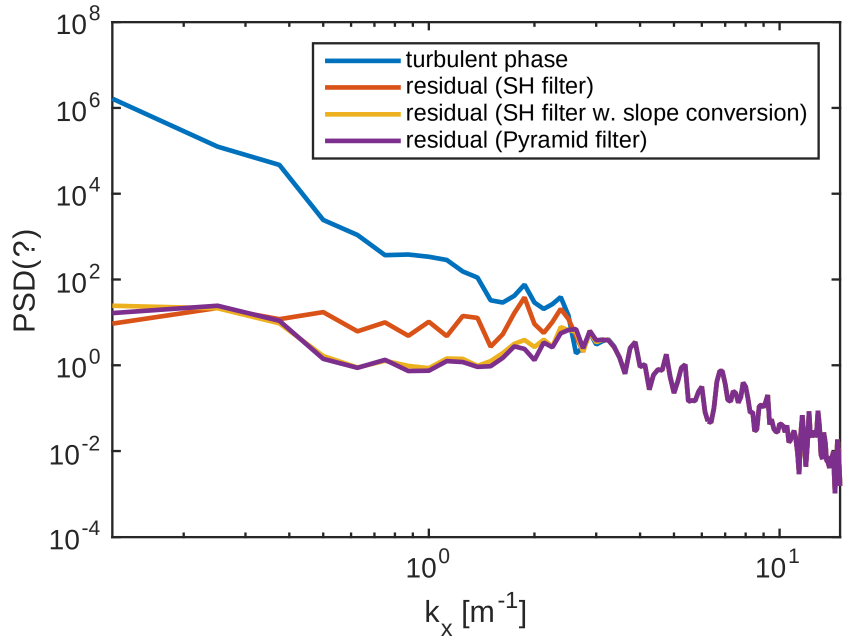
N correctable modes → **N log(N) operations**

- Increased computation speed.
- Insight into modal behaviour.
- WF error links directly to PSF shape



E2E simulations

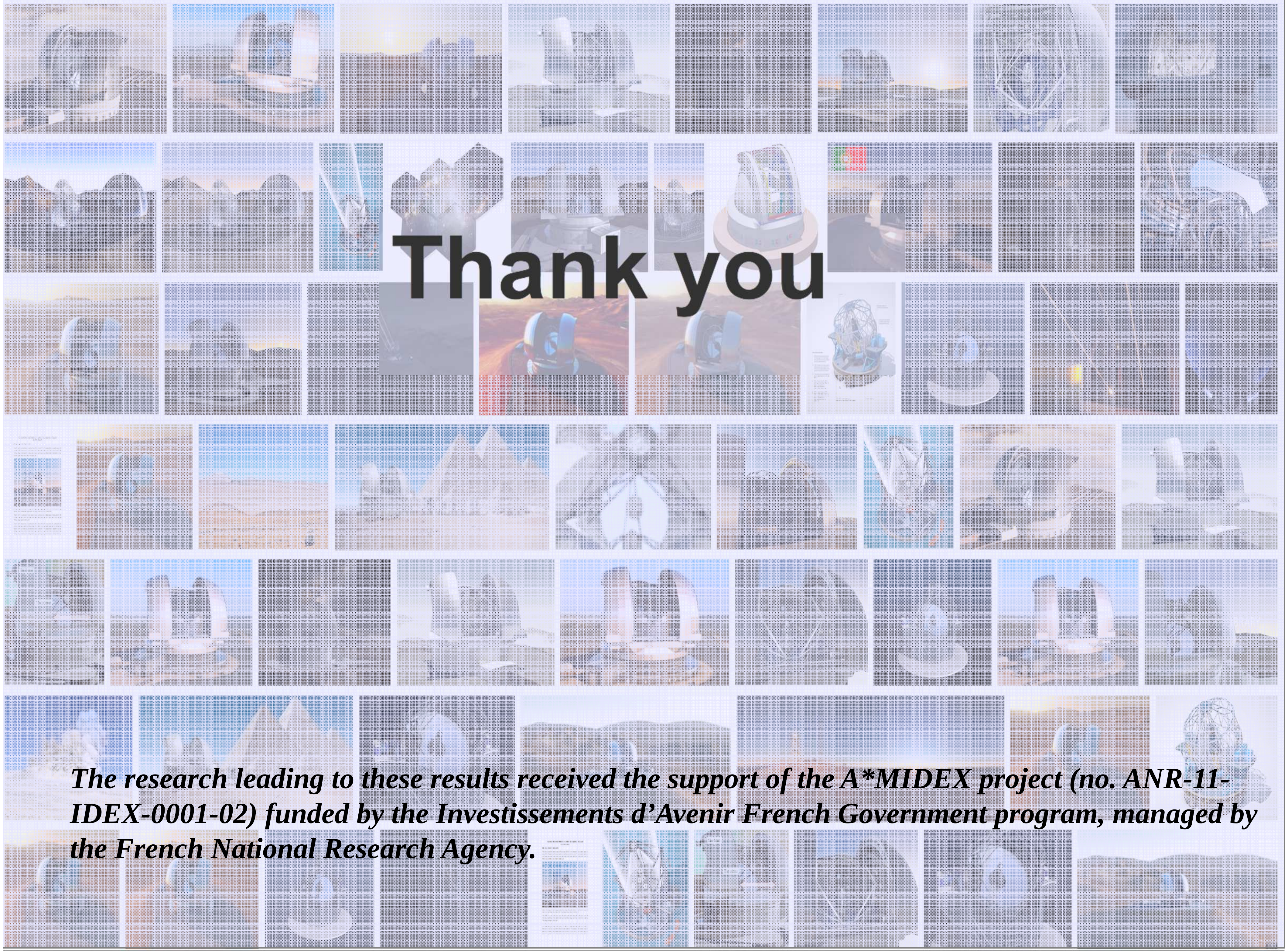




- C. Bond, et al, *Anti-aliasing wave-front reconstruction with Shack-Hartmann sensors,* Proc. of the AO4ELT4, Lake Arrowhead, CA, USA (2015)
- C. Bond et al, *Real-time WF reconstruction from pyramid signals in the Fourier domain*, Wavefront Sensing in the VLT/ELT era, Marseille, (September 2016)
- C. Bond, et al, *Iterative wave-front reconstruction in the spatial-frequency domain*, Optics Express, **25**(10), 11452-11465 (2017) https://doi.org/10.1364/OE.25.011452

Summary

- Using pyramids **increases the limiting magnitude**
- Use of time-progression models into the controller + Fourier modes control can **improve the rejection at small separations**
- **Aliasing** can be **further beaten down** using proper models
- Error PSD on complex exponentials => post-coronagraphic PSF
  - useful for post-processing?



Thank you

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